# PRAYAS

## JEE 2026

**Mathematics**

# Basic Maths

**Lecture -14**

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# Topics *to be covered*

A Inequalities involving Modulus

B Problem Practice

# Homework Discussion

**QUESTION**

Solve : $\log_{\frac{1}{x}}\left(\frac{2(x-2)}{(x+1)(x-5)}\right) \geq 1$

TAH 02 $\frac{2(x-2)}{(x+1)(x-5)} > 0$

$x \in (-1,2) \cup (5,\infty)$ Ⓐ

Case ① If $\frac{1}{x} > 1 \Rightarrow \frac{1}{x} - 1 > 0 \Rightarrow \frac{1-x}{x} > 0$

$\Rightarrow \frac{x-1}{x} < 0 \Rightarrow x \in (0,1)$

$$\frac{2(x-2)}{(x+1)(x-5)} \geq \frac{1}{x}$$

$$\frac{2(x-2)}{(x+1)(x-5)} - \frac{1}{x} \geq 0$$

$$\frac{2x^2 - \cancel{4x} - x^2 + \cancel{4x} + 5}{x(x-1)(x-5)} \geq 0$$

$$\frac{x^2+5}{(x+1)(x-5)x} \geq 0 \longrightarrow \frac{1}{x(x+1)(x-5)} \geq 0$$

Always +ve

$x \in (-1,0) \cup (5,\infty)$

$\cap$ $x \in \phi$

Case ⑪ If $0 < \frac{1}{x} < 1$

$\frac{1}{x} > 0$ ✓ $\downarrow$ $x > 0$

$\frac{1-x}{x} < 0$ ✓ $\frac{x-1}{x} > 0$

$x \in (-\infty,0) \cup (1,\infty)$

$\cap$ $x \in (1,\infty)$

$$\frac{2(x-2)}{(x+1)(x-5)} \leq \frac{1}{x}$$

$$\frac{x^2+5}{x(x+1)(x-5)} \leq 0$$

$x \in (-\infty,-1) \cup (0,5)$

$\cap$ $x \in (1,5)$

$A \cap B$

$(1,5) \cup \phi = B$

$B = (1,5)$

$(-1,2) \cup (5,\infty)$

$\cap$

$x \in (1,2)$ Ans

## QUESTION [JEE Mains 2025 (8 April)]

TAH 06

The sum of the squares of the roots of $|x-2|^2 + |x-2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x-3| - 5 = 0$, is

- (A) 24
- (B) 26
- (C) 36
- (D) 30

Case① if $x-3 \geq 0 \Rightarrow x \geq 3$

$x^2 - 2x + 6 - 5 = 0$

$x^2 - 2x + 1 = 0$

$x = 1. \cap$

$x \in \phi$

Case(ii) if $x-3 < 0 \Rightarrow x < 3$

$x^2 + 2x - 6 - 5 = 0$

$x^2 + 2x - 11 = 0$

$x = \dfrac{-2 \pm \sqrt{48}}{2}$

$x = -1 \pm 2\sqrt{3} \Rightarrow x = \overset{\checkmark}{2\sqrt{3}-1}, \overset{\checkmark}{-2\sqrt{3}-1}$

$|x-2| = t$

$t^2 + t - 2 = 0$

$t = -2, 1$

$|x-2| = \cancel{-2}, 1$

$x - 2 = -1, 1$

$x = 1, 3$

Sum of squares of roots $= 1 + 9 + (2\sqrt{3}-1)^2 + (-2\sqrt{3}-1)^2$

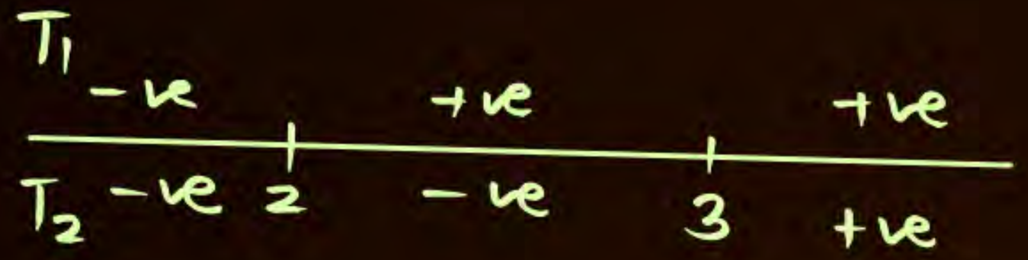
$= 10 + 13 - 4\sqrt{3} + 13 + 4\sqrt{3}$

$= 36$ Ans

Ans. C

The number of real roots of the equation $x|x-2|+3|x-3|+1=0$ is:

- A 4
- B 3
- C 2
- D 1



$\frac{-1+\sqrt{33}}{2} \; \textcircled{<} \; 3$

$-1+\sqrt{33} \; \bigcirc \; 6$

$\sqrt{33} \; \textcircled{<} \; 7$

case ① if $x \le 2$

$-x(x-2)-3(x-3)+1=0$

$-x^2+2x-3x+10=0$

$-x^2-x+10=0$

$x^2+x-10=0$

$x=\frac{-1\pm\sqrt{41}}{2} = \cancel{\frac{-1+\sqrt{41}}{2}}, \frac{-1-\sqrt{41}}{2}$ ✓

$\boxed{x=\frac{-1-\sqrt{41}}{2}}$ Ans.

case ⑪ If $2<x<3$

$x(x-2)-3(x-3)+1=0$

$x^2-5x+10=0$

No real roots.

case ⑾ if $x \geq 3$

$x(x-2)+3(x-3)+1=0$

$x^2-2x+3x-9+1=0$

$x^2+x-8=0$

$x=\frac{-1\pm\sqrt{33}}{2} = \cancel{\frac{-1-\sqrt{33}}{2}}, \cancel{\frac{-1+\sqrt{33}}{2}}$

Ans. D

# QUESTION [JEE Mains 2024 (5 April)]

TAH 08

The number real solutions of the equations $x|\overset{T_1}{x+5}| + 2|\overset{T_2}{x+7}| - 2 = 0$ is

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | -ve | | -ve | | +ve |
| $T_2$ | -ve | -7 | +ve | -5 | +ve. |

**Case ①** if $x \le -7$

$$-x^2-5x-2x-14-2=0$$

$$x^2+7x+16=0$$

$D<0$

↓

No real roots.

**Case ⅱ** if $-7 < x < -5$

$$-x^2-5x+2x+14-2=0$$

$$-x^2-3x+12=0$$

$$x^2+3x-12=0$$

$$x=\frac{-3\pm\sqrt{57}}{2}$$

$$x=\frac{-3-\sqrt{57}}{2}, \cancel{\frac{-3+\sqrt{57}}{2}}$$

**Case ⅲ** if $x \geq -5$

$$x^2+5x+2x+14-2=0$$

$$x^2+7x+12=0$$

$$x=-4,-3.$$

$\cup$

$$x=-3,-4,\frac{-3-\sqrt{57}}{2}$$

Ans. 3

$$-7 \ \textcircled{<} \ \frac{-3-\sqrt{57}}{2} \ \textcircled{<} \ -5$$

$$-14 \ \bigcirc \ -3-\sqrt{57} \ \bigcirc \ -10$$

$$-11 \ \textcircled{<} \ -\sqrt{57} \ \textcircled{<} \ -7$$

$$\parallel$$

$$-7.....$$

## QUESTION [JEE Mains 2024 (5 April)]

TAH 09

The number of distinct real roots of the equation |x| |x + 2| – 5|x + 1| – 1 = 0 is

$$|\overset{T_1}{x}|\,|\overset{T_2}{x+2}|-5|\overset{T_3}{x+1}|-1=0$$

Hogayaa ↑

Nahi huaa ↓

|  |  | −2 |  | −1 |  | 0 |  |
|---|---|---|---|---|---|---|---|
| $T_1$ | −ve |  | −ve |  | −ve |  | +ve |
| $T_2$ | −ve |  | +ve |  | +ve |  | +ve |
| $T_3$ | −ve |  | −ve |  | +ve |  | +ve |

**Case ① if $x \le -2$**

$$-x\cdot-(x+2)+5(x+1)-1=0$$

$$x^2+2x+5x+4=0$$

$$x^2+7x+4=0$$

$$x=\cancel{\frac{-7+\sqrt{33}}{2}},\ \boxed{\frac{-7-\sqrt{33}}{2}}$$

**Case (II) if $-2<x<-1$**

$$-x(x+2)+5(x+1)-1=0$$

$$-x^2+3x+4=0$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x=4,-1$$

(rejected)

**case (III) if $-1\le x\le 0$**

$$-x^2-2x-5x-5-1=0$$

$$x^2+7x+6=0$$

$$x=\cancel{-6},-1$$

$$\boxed{x=-1}$$

**case (IV) if $x>0$**

$$x^2+2x-5x-5-1=0$$

$$x^2-3x-6=0$$

$$x=\frac{3\pm\sqrt{33}}{2}$$

$$\boxed{x=\frac{3+\sqrt{33}}{2}}$$

Ans. 3

## QUESTION [JEE Mains 2024 (8 April)]

TAH 11

The number of distinct real roots of the equation $|x + 1|\,|x + 3| - 4|x + 2| + 5 = 0$ is

Redo

Ans. 2

# Lo Karo Duvaadaar Practice!!

ReTry

1. $\log_5(x^2 - 3x + 3) > 0$

2. $\log_7[\log_5(x^2 - 7x + 15)] > 0$

3. $\log_{\left(\frac{1}{2}\right)}[\log_5(x^2 - 7x + 17)] > 0$

4. $\log_{\left(\frac{1}{2}\right)}(\log_5(\log_2(x^2 - 6x + 40))) > 0$

5. $\log_3[\log_5 \log_2(x^2 - 9x + 50)] > 0$

6. $\log_6\left(\frac{x-2}{6-x}\right) > 0$

7. $\log_{0.5}(x^2 - 5x + 6) > -1$

8. $\log_8(x^2 - 4x + 3) < 1$

$x^2-4x+3<8$ & $x^2-4x+3>0$

$x^2-4x-5<0$ $\quad (x-3)(x-1)>0$

$(x-5)(x+1)<0$ $\quad x\in(-\infty,1)\cup(3,\infty)$

$x\in(-1,5)$

$x\in(-1,1)\cup(3,5)$

9. $\log_{\left(\frac{1}{4}\right)}\left(\frac{35-x^2}{x}\right) \geq -\frac{1}{2}$ & $\frac{35-x^2}{x}>0$

$\frac{35-x^2}{x} \leq (1/4)^{-1/2} = 2.$

$$\frac{35-x^2}{x} \le 2$$

$$2+\frac{x^2-35}{x} \ge 0$$

$$\frac{x^2+2x-35}{x} \ge 0$$

$$\frac{(x+7)(x-5)}{x} \ge 0$$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -7 & & 0 & & 5 & \end{array}$$

$$x \in [-7,0) \cup [5,\infty)$$

$$\frac{35-x^2}{x} > 0$$

$$\frac{(x-\sqrt{35})(x+\sqrt{35})}{x} < 0$$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -\sqrt{35} & & 0 & & \sqrt{35} & \end{array}$$

$$x \in (-\infty,-\sqrt{35}) \cup (0,\sqrt{35})$$

$$x \in [-7,-\sqrt{35}) \cup [5,\sqrt{35})$$

③ $\log_{\frac{1}{2}}\left(\log_5(x^2-7x+17)\right)>0$

$\log_5(x^2-7x+17)<1$

$x^2-7x+17<5$

$x^2-7x+12<0$

$(x-4)(x-3)<0$

$x\in(3,4)$

& $\log_5(x^2-7x+17)>0$

$x^2-7x+17>1$

$x^2-7x+16>0$

$\downarrow$ $D<0,\ a>0$

$x\in R$

& $x^2-7x+17>0$

$\downarrow$ $D<0,\ a=1$

$x\in R$

$\cap$

$x\in(3,4)$

$$\log_3\left(\log_5\left(\log_2(x^2-9x+50)\right)\right) > 0$$

$$\log_5 \log_2(x^2-9x+50) > 3^0 = 1$$ ,

(No Need) (No Need)

$$\log_5\left(\log_2(x^2-9x+50)\right) > 0, \quad \log_2(x^2-9x+50) > 0$$

& $x^2-9x+50>0$

(No Need)

$$\log_2(x^2-9x+50) > 5.$$

$$x^2-9x+50 > 2^5 = 32$$

$$x^2-9x+18 > 0$$

$$(x-6)(x-3) > 0$$

$$x \in (-\infty,3) \cup (6,\infty)$$

# Answers

1. $(-\infty, 1) \cup (2, \infty)$
2. $(-\infty, 2) \cup (5, \infty)$
3. $(3, 4)$
4. $(2, 4)$
5. $(-\infty, 3) \cup (6, \infty)$
6. $(4, 6)$
7. ~~$(1, 4)$~~ $(1, 2) \cup (3, 4)$
8. $(-1, 1) \cup (3, 5)$
9. $(-1, 0) \cup (5, \infty)$

⑦ $\log_{0.5}(x^2-5x+6) > -1$

$x^2-5x+6 < (0.5)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$

$x^2-5x+4<0$

$(x-1)(x-4)<0$

$x \in (1,4)$

$x^2-5x+6>0$

$(x-2)(x-3)>0$

$x \in (-\infty,2) \cup (3,\infty)$

$\cap$

$x \in (1,2) \cup (3,4)$

## Home Challenge-05

If the value of x which satisfies the equation $2\log_3\sqrt{3^{1-x}+2} = 1 + \log_3(4\cdot 3^x - 1)$ is given by, $1 - \log_3 k$, then find the value of k. [Ans. 4]

$$\log_3 \sqrt{3^{1-x}+2}^{\,2} = \log_3 3 + \log_3(4\cdot 3^x - 1)$$

$\sqrt{x^2} = |x|$

$\sqrt{x}^2 = x$

$$\log_3(3^{1-x}+2) = \log_3(3\cdot(4\cdot 3^x - 1))$$

$$3^{1-x}+2 = 12\cdot 3^x - 3$$

$$\frac{3}{3^x}+2 = 12\cdot 3^x - 3$$

let $3^x = t$ $\quad \frac{3}{t}+2 = 12t-3$

$$12t^2-5t-3=0$$

$$12t^2-9t+4t-3=0$$

$$(3t+1)(4t-3)=0$$

$$t = -1/3,\ 3/4$$

$$3^x = \cancel{-1/3},\ 3/4$$

$$3^x = 3/4$$

$$\log_3 3^x = \log_3 (3/4)$$

$$x = 1 - \log_3 4$$

$$\boxed{k = 4}$$

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

**QUESTION**

Tah01

The number of solutions of the equation

$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0, x > 0$, is :

Ans. 1

**Question**

Tan02

Find the integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x-2\right|-\left|\log_3 x-2\right|=2$.

[Ans. 9]

$|2\log_3 x-2|-|\log_3 x-2|=2$

let $\log_3 x=t$ $\quad 2|t-1|-|t-2|=2$

## QUESTION

$$(x-1)\,|x^2-4x+3|+2x^2+3x-5=0$$

**case①** if $x^2-4x+3\geq 0 \Rightarrow (x-1)(x-3)\geq 0$

$x\in(-\infty,1]\cup[3,\infty)$

$$(x-1)\,(x^2-4x+3)+2x^2+5x-2x-5=0$$

$$(x-1)^2\,(x-3)\quad+(2x+5)\,(x-1)=0$$

$$(x-1)\left[(x-1)(x-3)+2x+5\right]=0$$

$\boxed{x=1}$ or $x^2-4x+3+2x+5=0$

$x^2-2x+8=0$ → No real roots.

$D<0$

$\boxed{x=1}$

**case (II)** if $x^2-4x+3<0$

$x\in(1,3)$

$$-(x-1)(x^2-4x+3)+(2x+5)\,(x-1)=0$$

$$(x-1)\,(-x^2+4x-3+2x+5)=0$$

$$-x^2+6x+2=0$$

$x=1$ or $x^2-6x-2=0$

$$x=\frac{6\pm\sqrt{44}}{2}$$

$$x=\cancel{3+\sqrt{11}},\ \cancel{3-\sqrt{11}}$$

$x\notin\phi$

## QUESTION

$|x^2 + 4x + 3| + 2x + 5 = 0$ Tah03

# QUESTION

$$|x - |4 - x|| = 2x + 4$$

$A \cup \phi = A$

$A \cap \phi = \phi$

$|4-x| = |-(x-4)| = |-1|\,|x-4| = |x-4|$

$|x-y| = |y-x|$

$$\big|x - |x-4|\big| = 2x+4$$

case① if $x-4 \geq 0 \Rightarrow x \geq 4$

$|x-x+4| = 2x+4$

$4 = 2x+4$

$\cancel{x=0}$

⇓

Nosoln

$x \in \phi$

case⑪ if $x-4<0 \Rightarrow x<4 \longrightarrow \big|x-(-(x-4))\big| = 2x+4$

$|x+x-4| = 2x+4$

$|2x-4| = 2x+4$

case ⓐ if $2x-4 \geq 0$

$x \geq 2$

$2x-4 = 2x+4$

$-4 = 4$

(N.P)

case ⓑ if $2x-4<0$

$x<2$

$-2x+4 = 2x+4$

$x=0$

UNION

$x=0$

$\cap$

$x=0$ Ans

**QUESTION**

$$|x^2 - 3|x| + 2| = x^2 - 2x$$

Case ① If $x \geq 0$

$$|x^2 - 3x + 2| = x^2 - 2x$$

Case ⓐ If $x^2 - 3x + 2 \geq 0$

$(x-1)(x-2) \geq 0$

$x \in (-\infty, 1] \cup [2, \infty)$

$x^2 - 3x + 2 = x^2 - 2x$

$\boxed{x = 2}$ ✓

Case ⓑ $x^2 - 3x + 2 < 0$

$x \in (1, 2)$

$-(x^2 - 3x + 2) = x^2 - 2x$

$-x^2 + 3x - 2 = x^2 - 2x$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$(2x-1)(x-2) = 0$

$x = \cancel{2, 1/2}$

$\boxed{x = 2}$

Case (II) If $x < 0$

$$|x^2 + 3x + 2| = x^2 - 2x$$

Case ⓐ $x^2 + 3x + 2 \geq 0$

$(x+1)(x+2) \geq 0$

$x \in (-\infty, -2] \cup [-1, \infty)$

$x^2 + 3x + 2 = x^2 - 2x$

$5x = -2$

$x = -2/5$

Case ⓑ $x^2 + 3x + 2 < 0$

$x \in (-2, -1)$

$-x^2 - 3x - 2 = x^2 - 2x$

$2x^2 + x + 2 = 0$

$D < 0$

No real roots.

$\boxed{x = -2/5}$

final Ans: $2, \frac{-2}{5}$.

# Inequalities Involving Modulus

P₆: $|x| \ge |y| \Leftrightarrow x^2 \ge y^2$

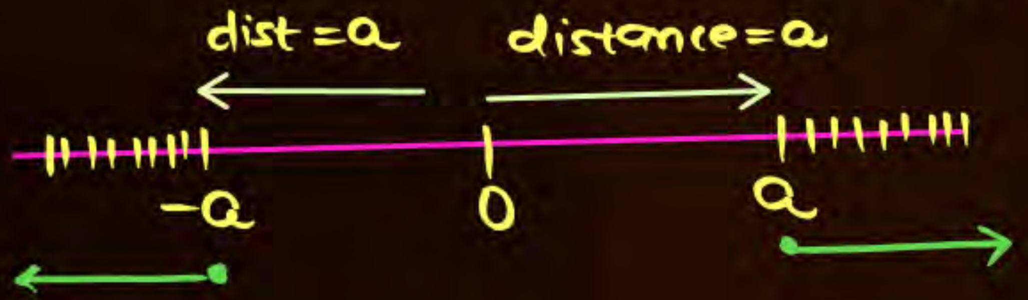
$|x| \geq |y|$ — Both sides non-ve

S.B.S $x^2 \geq y^2$

$(x-y)(x+y) \geq 0$

P₇: $|x| \ge a, a \in R^+$

$x \leq -a$ or $x \geq a$



$|x| > a$, $a \in R^+$

$x > a$ or $x < -a$

P₈: $|x| \le a, a \in R^+$

$-a \leq x \leq a$

$|x| < a$, $a \in R^+$

$-a < x < a$

## QUESTION

Solve following inequalities:

(i) $|x| \geq 2 \longrightarrow x \geq 2$ or $x \leq -2 \Rightarrow x \in (-\infty, -2] \cup [2, \infty)$

(ii) $|x| < 5 \longrightarrow -5 < x < 5 \Rightarrow x \in (-5, 5)$

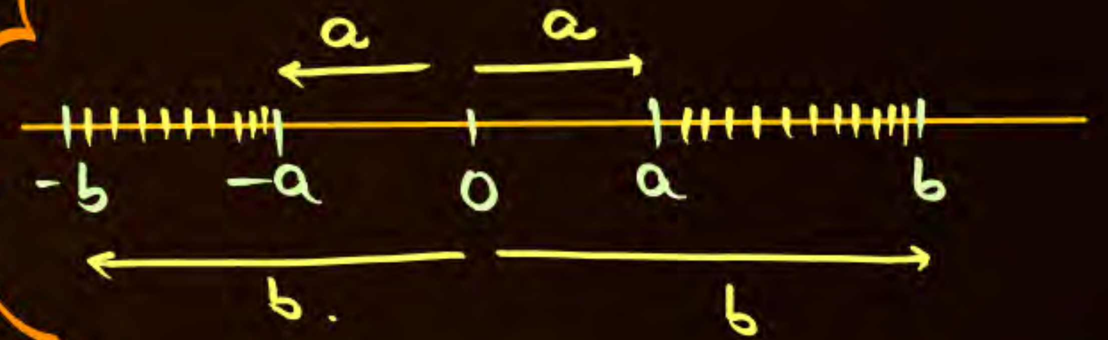
(iii) $3 < |x| \leq 7 \longrightarrow x \in [-7, -3) \cup (3, 7]$

(iv) $|x| + 5 \geq 0 \longrightarrow |x| \geq -5 \longrightarrow x \in R$

(v) $3|x| + 2 < 0 \longrightarrow |x| < -2/3 \longrightarrow x \in \phi$

(vi) $3|x| + 2 \leq 0 \longrightarrow |x| \leq -2/3 \longrightarrow x \in \phi$

$a \leq |x| \leq b \quad , a, b \in R^+$



$x \in [-b, -a] \cup [a, b]$

# QUESTION

If $|x - 3| \geq 2$ then

[✓ A] $x \geq 5$ or $x \leq 1$

B $x \geq 5$ or $x \geq 1$

C $x \leq -5$ or $x \geq 1$

D $x \leq -1$ or $x \geq 5$

$$|x| \geq a \begin{cases} x \geq a \\ \text{or} \\ x \leq -a \end{cases}$$

$x - 3 \geq 2$ or $x - 3 \leq -2$

$x \geq 5$ or $x \leq 1$

## QUESTION

If $|3 - 2x| \leq 7$ then

[✓ A] $-2 \leq x \leq 5$

B $-5 \leq x \leq 2$

C $-2 \leq x \leq 2$

D $-5 \leq x \leq -2$

$$|x| \leq a$$
$$-a \leq x \leq a$$

$$|3-2x| \leq 7$$
$$-7 \leq 3-2x \leq 7$$
$$-10 \leq -2x \leq 4$$
$$\frac{-10}{-2} \geq x \geq \frac{4}{-2}$$
$$5 \geq x \geq -2$$
$$-2 \leq x \leq 5.$$

$$|-2| = |2|$$

$$|-x| = |x|$$

$$|4-x| = |-(x-4)|$$

$$= |x-4|$$

**QUESTION**

If $|x-2| \le x^2 - x - 1$ then find the possible set of all values of x.

$-(x^2-x-1) \le x-2 \le x^2-x-1 \longrightarrow$ nahi karnaa

$|(x-2)| \le x^2-x-1$

$\left(\text{Any real}\right)^{\text{Even power}} \ge 0$

NO

case① if $x-2 \ge 0 \Rightarrow x \ge 2$

$x-2 \le x^2-x-1$

$x^2-2x+1 \ge 0$ $\quad \cap \quad x \ge 2$.

$(x-1)^2 \ge 0$

$\downarrow$

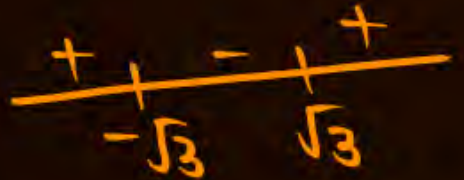
$x \in R$

case(ii) If $x-2<0 \Rightarrow x<2$

$-(x-2) \le x^2-x-1$

$\cancel{-x}+2 \le x^2\cancel{-x}-1$

$x^2-3 \ge 0$



$\rightarrow x \in (-\infty,-\sqrt{3}] \cup [\sqrt{3},\infty)$

$\cap$

$x \in (-\infty,-\sqrt{3}] \cup [\sqrt{3},2)$

yaaha kar sakte hai

$|x^2-5x+6| \le x^2+x+1$ — always +ve — $D<0, a>0$

$-(x^2+x+1) \le x^2-5x+6 \le x^2+x+1$

$2x^2-4x+7 \ge 0$ $\qquad 6x \ge 5$

$D<0$ $\qquad x \ge 5/6$

$\downarrow$

always +ve

$\downarrow$

$x \in R$ $\quad \cap \quad$ $x \in [5/6,\infty)$

final ans: $x \geq 2 \cup x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, 2)$

✓

$$\underbrace{x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)}$$

# Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

## Home Challenge-06

If $x = \alpha$ is the solution of the equation $|2 + \log_2 7x| - \log_2(x - 1) = 5$, then find the value of $(65)^{\frac{1}{3}\log_{\alpha^2+1}\alpha}$.

[Ans. 2]

# Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

**QUESTION**

(KTK 1)

Solve for x : $3|x^2 - 4x + 2| = 5x - 4$

Ans. $x = 2, 5$

**QUESTION**

(KTK 2)

The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is :

- **A** 4
- **B** 3
- **C** 5
- **D** 6

Ans. B

**QUESTION**

KTK 03

Solve for x :

$$\log 4+\left(1+\frac{1}{2x}\right)\log 3=\log\left(\sqrt[x]{3}+27\right)$$

Ans. $x \in \phi$

QUESTION

KTK 04

If $(x_1, y_1)$ and $(x_2, y_2)$ are the solution of the system of equation.

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1,$

then show that the value of $\log_{30}(x_1 y_1 x_2 y_2) = 12.$

QUESTION

KTK 05

The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

Ⓐ $\log_2 14$

Ⓑ $\log_2 11$

Ⓒ $\log_2 12$

Ⓓ $\log_2 13$

Ans. B

QUESTION

KTK 06

Evaluate $\dfrac{\left((64)^{\frac{1}{\log_5 8}}+2^{\frac{2}{\log_{\sqrt{5}} 2}}\right)\left((\sqrt{11})^{\frac{2}{\log_{25} 11}}-(64)^{\log_8 \sqrt{5}}\right)}{300}$

Ans. 2

**QUESTION**

**KTK 09**

Simplify : $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2}\frac{1}{10+2\sqrt{21}}$

Ans. 6

# Homework From Module

**Prarambh (Topicwise)** : Q1 to Q17

**Prabal (JEE Main Level)** : Q1 to Q7

# Solution to Previous TAH

## QUESTION [JEE Mains 2021]

TAH 01

The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is

# Lec-13

Tah-01 $\log_4(x-1) = \log_2(x-3)$

$\log_{2^2}(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1) = 2\log_2(x-3)$

$\Rightarrow (x-1) = (x-3)^2$

$\Rightarrow x-1 = x^2+9-6x$

$\Rightarrow x^2-7x+10=0$

$\Rightarrow (x-5)(x-2)=0$

$\Rightarrow x = 5, 2 \times$

$\boxed{x=2}$

no. of sol$^n$ = 1 An

QUESTION

TAH 03

$$\left|\left(\frac{x^2-6x+8}{x^2-4x+3}\right)\right| = -\left(\frac{x^2-6x+8}{x^2-4x+3}\right)$$

**TAH 03:-**

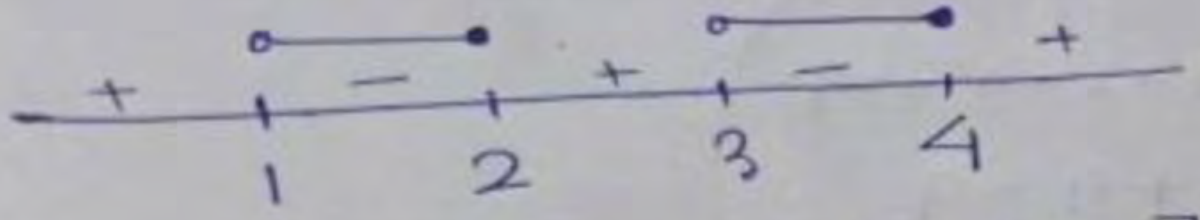
$$\underset{|f(x)|}{\underset{\shortparallel}{\left|\left(\frac{x^2-6x+8}{x^2-4x+3}\right)\right|}} = \underset{-f(x)}{\underset{\shortparallel}{-\left(\frac{x^2-6x+8}{x^2-4x+3}\right)}}$$

$$f(x) \leq 0$$

$$\Downarrow$$

$$\frac{x^2-6x+8}{x^2-4x+3} \leq 0$$

$$\frac{(x-2)(x-4)}{(x-1)(x-3)} \leq 0$$



$$\therefore x \in (1, 2] \cup (3, 4]$$

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ASSAM

QUESTION

TAH 04

If $|3x - 2| + x = 11$ then x is

A $13/4$

B $9/2$

C $-9/2$

D $-13/4$

n 4  $|3x-2|+x = 11$

Case ① [redacted]

if $3x-2 \geq 0$  $x \geq 2/3$

$3x-2+x = 11$

$4x-2 = 11$

$2x = \frac{11}{2}+1$

$x = \frac{13}{4}$  $\cap$

**sakshi sahu**
**from mp**

Case 2: if $3x-2 < 0 \Rightarrow x < 2/3$

$-3x+2+x = 11$

$-2x+2 = 11$

$(x-1) = -\frac{11}{2}$

$x = -\frac{9}{2}$  $\cap$

$x = \frac{13}{4}$  $\cup$  $x = \frac{-9}{2}$

$x = \frac{13}{4}, -\frac{9}{2}$  (A),(B)

5 [redacted]

Sourav Kalita
ASSAM

**TAH 04:**

$$|3x-2|+x=11$$

**Case I:** $3x-2 \geqslant 0 \Rightarrow x \geqslant 2/3$

$$3x-2+x=11$$

$$4x=13$$

$$\boxed{x=\frac{13}{4}}$$

$\cap$

**Case II:** $3x-2<0 \Rightarrow x<2/3$

$$-3x+2+x=11$$

$$-2x=9$$

$$\boxed{x=-\frac{9}{2}}$$

$x=13/4$ $\qquad$ $x=-9/2$

$\cup$

$\boxed{x=\frac{13}{4}}$ or $\boxed{x=\frac{-9}{2}}$ //

QUESTION

TAH 05

$|x-1|+|x-2|+|x-3|=9$ then x can be

(A) $-5$

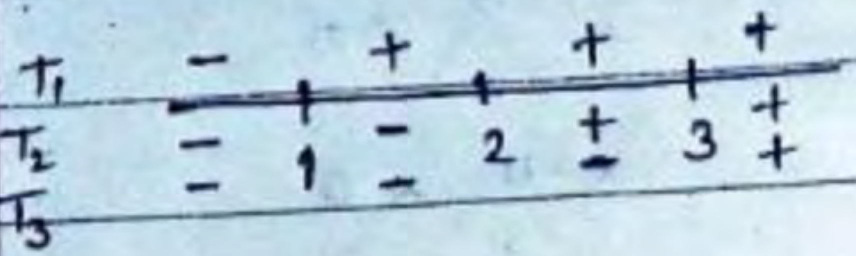
(B) $9$

(C) $-1$

(D) $5$

**sakshi sahu**
**from mp**

Tah 5 $\overset{T_1}{|x-1|}+\overset{T_2}{|x-2|}+\overset{T_3}{|x-3|}=9$



Case ①: if $x\leq 1$

$-(x-1)-(x-2)-(x-3)=9$

$-x+1-x+2-x+3-9=0$

$-3x-3=0$

$x=-1$ $\cap$

$x=-1$

Case ②: if $1<x\leq 2$

$(x-1)-(x-2)-(x-3)=9$

$\cancel{x}-1-\cancel{x}+2-x+3-9=0$

$-x-5=0$

$x=-5$ $\cap$

$\phi$

Case ③ if $2<x\leq 3$

$(x-\cancel{1})+\cancel{x}-\cancel{2}-\cancel{x}+\cancel{3}-9=0$

$x=9$ $\cap$

$\phi$

Case 4 if $x>3$

$x-1+x-2+x-3-9=0$

$3x-15=0$

$x=5$ $\cap$

$x=5$

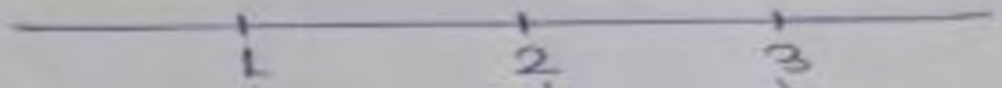
take union all 4 cases

$x=-1,5$ (c), (d) Ans

Sourav Kalita
ASSAM

## TAH 05 :

$$|\overset{T_1}{x-1}| + |\overset{T_2}{x-2}| + |x\overset{T_3}{-3}| = 9$$



| | | | | |
|---|---|---|---|---|
| $T_1 \longrightarrow$ | -ve | +ve | +ve | +ve |
| $T_2 \longrightarrow$ | -ve | -ve | +ve | +ve |
| $T_3 \longrightarrow$ | -ve | -ve | -ve | +ve |

**Case I :** If $x \leqslant 1$

$$-x+1-x+2-x+3=9$$
$$-3x=3$$
$$x=-1$$
$$\cap$$
$$x=-1$$

**Case II :** If $1 < x \leqslant 2$

$$x-1-x+2-x+3=9$$
$$-x+4=9$$
$$x=-5$$
$$\cap$$
$$x=\phi$$

**Case III :** $2 < x \leqslant 3$

$$x-1+x-2-x+3=9$$
$$x=9$$
$$\cap$$
$$x=\phi$$

**Case IV :** $x \geqslant 3$

$$x-1+x-2+x-3=9$$
$$3x-6=9$$
$$x=5$$
$$\cap$$
$$x=5$$

$\therefore$ Ans: $\boxed{x=-1, 5}$ ✓✓

**QUESTION [JEE Mains 2025 (7 April)]**

TAH 07

The number of real roots of the equation $x|x-2|+3|x-3|+1=0$ is:

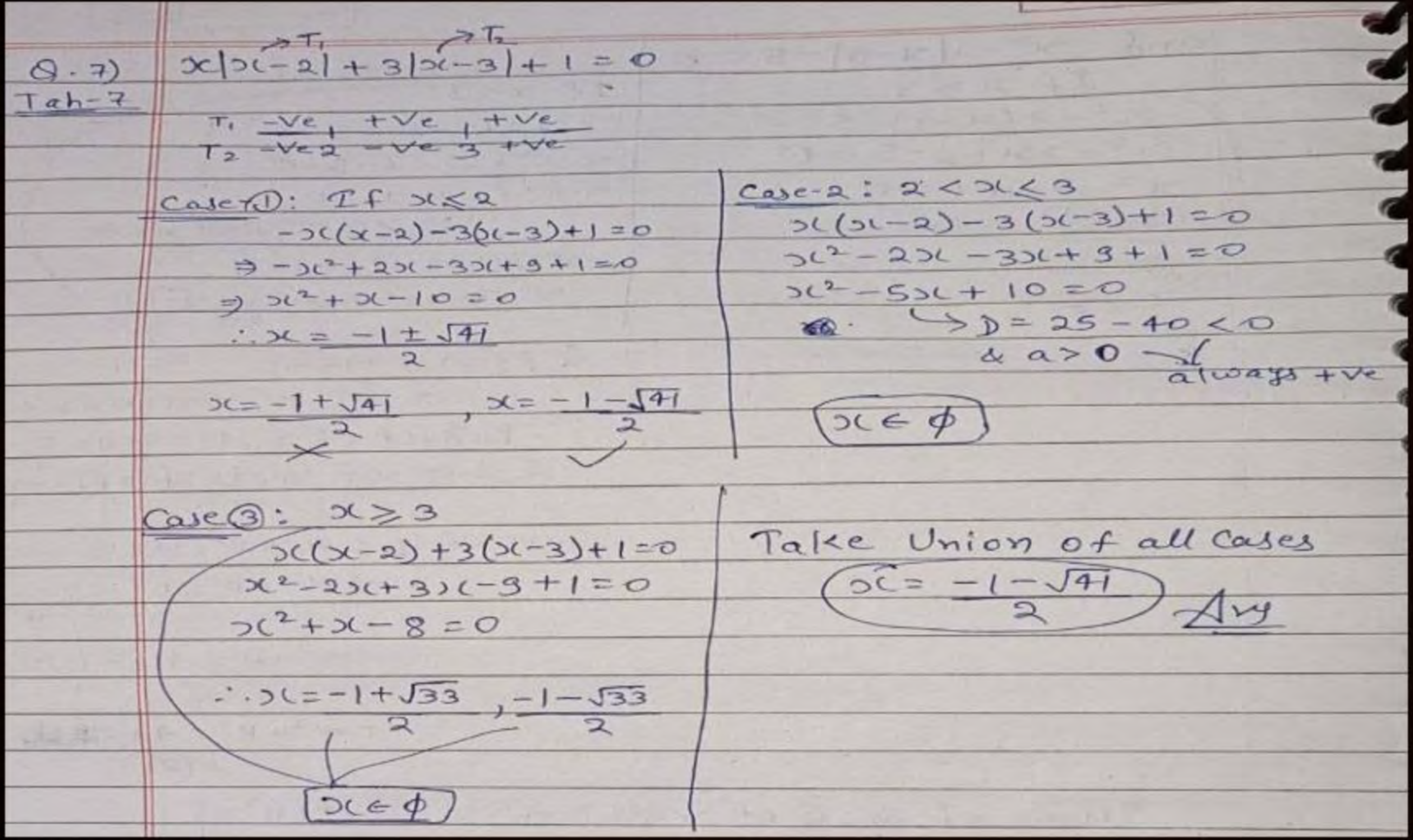
(A) 4

(B) 3

(C) 2

(D) 1

Ans. D



Q.7) Tah-7

$$x|\overset{\to T_1}{x-2}|+3|\overset{\to T_2}{x-3}|+1=0$$

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | $-ve$ | | $+ve$ | | $+ve$ |
| $T_2$ | $-ve$ | 2 | $-ve$ | 3 | $+ve$ |

Case-①: If $x\leq 2$

$$-x(x-2)-3(x-3)+1=0$$

$$\Rightarrow -x^2+2x-3x+9+1=0$$

$$\Rightarrow x^2+x-10=0$$

$$\therefore x=\frac{-1\pm\sqrt{41}}{2}$$

$$x=\frac{-1+\sqrt{41}}{2} \times \quad , \quad x=\frac{-1-\sqrt{41}}{2} \checkmark$$

Case-2: $2<x\leq 3$

$$x(x-2)-3(x-3)+1=0$$

$$x^2-2x-3x+9+1=0$$

$$x^2-5x+10=0$$

$\hookrightarrow D=25-40<0$

& $a>0 \to$ always +ve

$$\boxed{x\in\phi}$$

Case③: $x\geq 3$

$$x(x-2)+3(x-3)+1=0$$

$$x^2-2x+3x-9+1=0$$

$$x^2+x-8=0$$

$$\therefore x=\frac{-1+\sqrt{33}}{2},\ \frac{-1-\sqrt{33}}{2}$$

$$\boxed{x\in\phi}$$

Take Union of all cases

$$x=\frac{-1-\sqrt{41}}{2}$$ Ans

## QUESTION [JEE Mains 2024 (5 April)]

TAH 08

The number real solutions of the equations $x|x+5|+2|x+7|-2=0$ is

Ans. 3

## IAH 08:

$$x|\overset{T_1}{x+5}| + 2|\overset{T_2}{x+7}| - 2 = 0$$

$-7$ $\qquad$ $-5$

| | | | |
|---|---|---|---|
| $T_1 \rightarrow$ | $-ve$ | $-ve$ | $+ve$ |
| $T_2 \rightarrow$ | $-ve$ | $+ve.$ | $+ve.$ |

**Case I:** $x \leq -7$

$$x(-x-5)+2(-x-7)-2=0$$
$$-x^2-5x-2x-14-2=0$$
$$-x^2-7x-16=0$$
$$x^2+7x+16=0$$
$$D = 7^2-4(16) = -ve$$

Not possible.

**Case II:** $-7 < x \leq -5$

$$x(-x-5)+2(x+7)-2=0$$
$$-x^2-5x+2x+14-2=0$$
$$-x^2-3x+12=0$$
$$x^2+3x-12=0$$
$$D=57$$
$$x = \frac{-3 \pm \sqrt{57}}{2}$$

$\cap$

$$\boxed{x = \frac{-3-\sqrt{57}}{2}}$$

**Case III:** $x > -5$

$$x(x+5)+2(x+7)-2=0$$
$$x^2+5x+2x+14-2=0$$
$$x^2+7x+12=0$$
$$x^2+3x+4x+12=0$$
$$(x+3)(x+4)=0$$
$$x=-3,-4$$

$\cap$

$$\boxed{x=-3,-4}$$

$\cup$

$$x = -3, -4, \frac{-3-\sqrt{57}}{2}$$

$\therefore$ Answer: ③ ✓

## QUESTION [JEE Mains 2021]

TAH 10

The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is

Q.10) $(x+1)^2 + |x-5| = \frac{27}{4}$

Tah-10

**Case ①: $x \geq 5$**

$(x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2+2x+1+x-5 = 27/4$

$\Rightarrow 4x^2+12x-16 = 27$

$\Rightarrow 4x^2+12x-43 = 0$

$D = 144+688 = 832 > 0$

$x = \frac{-12 \pm \sqrt{832}}{8}$

$x = \frac{-12 \pm 8\sqrt{13}}{8}$

$\cap$

$x \in \phi$

**Case-②: $x < 5$**

$x^2+2x+1-x+5 = 27/4$

$4x^2+4x+24 = 27$

~~$2x(2x+$ $2x(2x+3)-1(2$~~

$4x^2+6x-2x-3 = 0$

$2x(2x+3)-1(2x+3) = 0$

$(2x+3)(2x-1) = 0$

$x = -3/2,\ 1/2$

$\cap$

$x = \frac{-3}{2}, \frac{1}{2}$

$\cup$

$$\boxed{x = -3/2, \tfrac{1}{2}}$$

$\therefore$ No of roots = ② Ans

# THANK YOU